The Role of Liquidity in Futures Market Innovations

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I characterize the optimal design of a new futures market (an innovation) by an exchange in the presence of market frictions. Futures markets am characterized by both the contract and the level of trader participation both can be determined by an exchange. A game in which exchanges simultaneously design markets is considered, and a particular equilibrium (not necessarily unique) is constructed. A game in which exchanges sequentially design markets (and incur design costs) is also considered and the (generically unique) equilibrium is constructed. The nature of equilibrium with multiple exchanges is discussed in these simultaneous and sequential settings, illustrating the role played by liquidity considerations both in market design and in the nature of competition between exchanges.

Financial markets have seen a great number of innovations over the last 25 years, and futures markets are no exception. It is important to understand which new securities are expected to be popular and, therefore, offered to the investing public. In the context of futures markets, the question becomes, Which new futures contracts (innovations) are likely to be designed and offered by exchanges? Clearly, any decision-maker thinking about opening a new security

market should consider not only the securities that investors would like to trade but also the securities that investors can already trade. A futures exchange's optimal innovation must therefore recognize the current structure of futures markets. In this article, I characterize the futures innovation most preferred by an exchange, considering both the contracts traders desire and the contracts already available. Innovation by multiple exchanges is also characterized, and the interaction of innovations on one another is described.

Far from being static, futures exchanges are constantly innovating contracts. Silber (1981) counts 52 contract innovations from 1960 through 1969 and 102 from 1970 through 1980. Miller (1986) calls the introduction of financial futures the "most significant financial innovation of the last twenty years" (p. 463). Moreover, not all new futures contracts are successful. Silber regards about one quarter to one third of the new contracts in the period 1960 through 1977 as "successes."

Contract design is often explained with a contract-specific approach, typically a case study describing the success or failure of a particular contract. Sandor (1973), for example, presents a case study of plywood futures, and Silber (1981) examines silver, gold, and GNMA futures. Success or failure of a new (or changed) contract in these studies generally hinges on some contract-specific quality, either [in the language of Black (1986)] a commodity characteristic (for example, storability or homogeneity) or a contract characteristic (for example, contract size or delivery specification). Case-by-case analysis seems unsatisfactory with respect to a general theory of contract innovation, although it often offers insight on individual contracts.

In this article, I emphasize the primary role of the hedger in the existence of a futures market, an approach traceable to Working (1953). In this view, a futures market owes its existence to the demand generated by hedgers. Although, to some extent, hedgers may take positions that offset each other, a futures market normally must attract more liquidity (in the form of additional traders) to become truly successful. Thus, a successful futures market displays two qualities: a contract providing hedgers with a high-quality (low-residual-risk) hedge and a liquid market. Telser (1981) argues that liquidity is the key difference between futures and forward markets, suggesting "the demand for a fungible financial instrument traded in a liquid market is necessary for the creation of an organized futures market" (p. 8).

The hedger's need for both a good hedge and a liquid market is sometimes framed as a trade-off between a high-quality hedge in an illiquid contract and a lower-quality hedge in a liquid contract. Black (1986) calls use of a lower-quality hedge in a liquid contract "cross hedging" and argues that certain contracts failed because of the pres-

ence of a competing cross-hedge. Working **(1953)** gives the history of the short-lived North Pacific Coast wheat futures contract. Prices of Pacific Northwest, a soft wheat, were only loosely related to prices of the hard wheat traded in Kansas City and Chicago, giving "a very imperfect hedge for soft wheat in the Pacific Northwest" (p. **337).** Although the Chicago Board of Trade introduced a Pacific Northwest wheat contract in 1950, Pacific Northwest hedgers opted to trade hard wheat futures, which offered an imperfect but liquid hedge, rather than switch to the new soft wheat future, a closer but less liquid hedge. (As a result, the new contract was never successful.) Similar stories can be told for grain sorghums [better hedged with corn: Hieronymus (1977)], 90-day commercial paper [better hedged with Treasury bills: Cornell (1981)], barley [better hedged with corn: Gray (1970)], and flour [no futures market developed; better hedged with wheat: Gray (1970)]. Silber (1981) warns about competing against an exchange with established liquidity, an advantage "usually too much to overcome" (p. 132).

The idea of contract choice tied together with liquidity is central to this article. Exchanges are taken to be entrepreneurial entities that design markets in a world of imperfect liquidity. Markets are characterized by both the contract offered and the number of traders participating. There seem to be two possible natural objectives for the exchange. In Silber (1981) and Black (1986), exchanges maximize transaction volume. In Duffie and Jackson (1989), exchanges maximize transaction volume; since exchanges charge a (preset) fee per transaction, they are also maximizing their own revenues. In this article, exchanges maximize their own revenues. Revenues arise, however, by charging traders one (endogenous) market entry fee, in contrast to Duffie and Jackson's fee per transaction.

Optimal market innovation is a sensible question only in an incomplete-markets setting. The model features incomplete markets in which market entry costs impede full risk sharing. Traders participate either to hedge risk from an uncertain endowment ("hedgers") or to be compensated for acting as risk sharers ("investors"). Friction arises in two ways: investors are constrained to enter only one market, and must pay a fee (which may vary across markets) to enter at all. Investors are thus forced to specialize in a single market (in particular, they cannot "spread" between markets). Of course, investors are most interested in entering a market with high demand for their services (substantial hedging). Hedgers are allowed to trade in all markets without incurring transaction costs. Liquidity of a market (market depth) is found to depend both on the number of traders in that market and on the presence of markets with similar contracts.

The fee structure lends itself to the following interpretation. Inves-

tors can be thought of as exchange members: their normal business is to provide liquidity services to hedgers. The entry fee can be interpreted as the price of a seat on the exchange. In contrast, hedgers are not normally in the business of providing liquidity (and may in fact wish to trade only occasionally). In the spirit that a successful contract must attract hedgers' business, hedgers face no entry fees in this model.

Each exchange may design one market, selecting both the contract traded and the number of investors entering (implicitly setting an entry fee) to maximize its revenues. (In the above interpretation, exchanges maximize the total value of their seats.) In contrast, exchanges in Duffie and Jackson select only a contract and try to maximize volume (and revenues). Furthermore, with the same traders participating in each market, Duffie and Jackson have no variation of liquidity across markets. Anderson and Harris (1986) present a model of sequential innovation in which exchanges, faced with uncertain demand for a contract, select only the time to begin innovation.

An exchange's optimal innovation is found to match a common intuition: contracts are designed to service the greatest possible residual hedging demand, after accounting for hedging that can be done in other markets. An exchange's optimal innovation is not necessarily Pareto-optimal (even in the case of a single exchange), since the exchange's objective is to collect fees from investors, although not from hedgers. They therefore tend not to open markets where hedges balance, even though potential volume may be high. With simultaneous innovation, each exchange tries to find its own niche, and similar contracts are designed only if the hedging demand for them is large. In equilibrium, similar contracts require high liquidity in the form of high trader participation. With sequential innovation, exchanges that move early choose contracts to serve the highest hedging demand, and make sure liquidity is high enough to deter potential competitors. In equilibrium, contracts are orthogonal, and the contracts serving the most hedging demand have the highest liquidity.

The article is structured as follows. In Section 1, I model trading, involving a continuum of hedgers and a continuum of investors. In Section 2, I describe equilibrium pricing, taking the set of contracts and investor entry as given. In Section 3, I endogenize the contract and investor entry. An exchange chooses both of these ("innovates") with the objective of maximizing the total entry fees collected in their market (taking the other markets into account). It is shown that the innovation chosen may not be Pareto-efficient. In Section 4, I consider a game with exchanges simultaneously innovating. Equilibrium is not necessarily unique; one particular equilibrium is constructed. Interaction of markets and the nature of competition between

exchanges are discussed. In Section **5,** I consider a game with exchanges sequentially innovating. Here, exchanges incur a fixed cost when innovating; some exchanges may therefore not innovate at all. The (generically unique) equilibrium is constructed. The effect of competition and the threat of competition between exchanges is discussed. I conclude in Section 6. Proofs appear in the Appendix.

1. Trading

There are T trading periods in the model; at the end of each period, uncertainty is resolved, and random endowments are received. Uncertainty in period t is characterized as an r-dimensional standard normal random variable $z_t \sim N(0, I)$, independent across time. There are m organized markets $(m \leq r)$, indexed by i. A *futures contract* D_i is defined as an r-dimensional unit vector. Contract D_i pays off $D_i^T z_t$ in period t, with $D_i^T z_t \sim N(0, 1)$.

There are two types of traders: hedgers and investors. Traders have exponential utility over final wealth; for simplicity, all traders have the same absolute risk aversion γ. There is a mass of H hedgers; hedger h receives random endowment $E_{bt}^T z_t$ in period t, where E_{bt} is a known r-dimensional vector. (Because demand for risky assets is independent of wealth under exponential utility, adding constant endowments has no effect on any results.) Let $E_t = \int_{b=0}^{H} E_{bt} \, db$ represent aggregate endowment for period t. Each hedger may enter all m markets each period to trade, incurring no cost of entry.

Investors have no random endowment; they act solely as risk sharers. There is a potentially unlimited mass of investors. Investors are constrained to trade either in one market (for all T periods) or not at all. If we interpret investors as exchange members, then physical presence at the exchange and the cost of specializing in a market provide barriers to trading in more than one market. (Because hedgers face endowment risk, the costs of trading in multiple markets are assumed to be small enough relative to the hedging risk to be treated as fixed costs not affecting hedgers' decisions.) Investors choose which, if any, market to enter and trade in before period 1 begins. An investor entering market i incurs a one-time fee (membership cost) of k_i. Since the set of entered traders is the same each period, (any measure of) market liquidity should be independent of time.

Traders all act as price-takers and submit demands each period. Hedgers submit demands to each market, whereas investors submit demands to the single market (if any) they entered. Demands in period t may depend on p_t, that period's vector of futures prices. No restrictions on short sales are made. A market-clearing price vector is chosen each period, and demands are fulfilled at that price. Con-

tracts traded may be considered payable (or collectible) when uncertainty is revealed and endowments are paid.

Traders face mean-variance maximization problems each period because utility is exponential, and uncertainty is normal and independent across periods. Hedger h selects an m-dimensional vector $\theta_{bt}(p_t)$ of demands for each period t to maximize

$$\max_{\theta_{bt}} - p_t^{\mathsf{T}}\theta_{bt} - (\gamma/2)(\theta_{bt}^{\mathsf{T}}D^{\mathsf{T}}D\theta_{bt} + 2\theta_{bt}^{\mathsf{T}}D^{\mathsf{T}}E_{bt} + E_{bt}^{\mathsf{T}}E_{bt}), \quad (1)$$

where D is the $r \times m$ matrix whose ith column is D_i. The first term in (1) is the mean, and the last terms are the variance of the hedger's overall position (futures plus endowment) for period t.

An investor who has entered market i selects a scalar demand $x_{it}(p_{it})$ for each period t to maximize

$$\max_{x_{it}} - p_{it}x_{it} - (\gamma/2)x_{it}^2. \quad (2)$$

Investors make their entry decisions before period 1. This implies investor entry until the certainty equivalent value for an investor trading in market i equals the entry cost k_i. Let n_i be the mass of investors in market i, and N be the $m \times m$ diagonal matrix with entries n_i. Define a *contract pair* (D_i, n_i) as a futures contract and the mass of investors in market i. Define a *market structure as* a set of contract pairs.

2. Trading Equilibrium

Define a *trading equilibrium* for a market structure $\{(D_i, n_i) : 1 \leq i \leq m\}$ as a vector of market-clearing prices, such that all traders are submitting optimal demands. Trader problems (1) and (2) are maximizations of quadratics; their solutions are as follows.

Lemma 1. *The solution to hedger h's problem (1) is*

$$\hat{\theta}_{bt} = (D^{\mathsf{T}}D)^{-1}(-D^{\mathsf{T}}E_{bt} - \gamma^{-1}p_t),$$

if $D^T D$ is nonsingular. The net demand across hedgers is then

$$\hat{\Theta}_t = (D^{\mathsf{T}}D)^{-1}(-D^{\mathsf{T}}E_t - \gamma^{-1}Hp_t).$$

If $D^T D$ is singular, some contract is redundant; an arbitrary set of prices may lead to an arbitrage opportunity, with demands not well defined.

Lemma 2. The solution to an investor in market i's problem (2) is

$$\hat{x}_{it} = -\gamma^{-1}p_{it}.$$

The net demand across investors is

$$\hat{X}_t = -\gamma^{-1}Np_t.$$

Because futures contracts are in zero net supply, the requirement for market clearing in period t is

$$\Theta_t(p_t) + X_t(p_t) = 0.$$

This can be solved for equilibrium prices. I also wish to characterize market liquidity: in particular, market depth (the order size required to change prices a given amount). It is convenient to first characterize market illiquidity. Define *market shallowness* (the "inverse" of depth) as the amount prices change for some exogenous demand shock. To be precise, for an exogenous demand shock vector u_i, market clearing requires

$$\Theta_t(p_t) + X_t(p_t) + u_t = 0.$$

Market shallowness is the $m \times m$ matrix $\partial p_t / \partial u_t$, implied from market clearing. Shallowness is characterized by a matrix because an exogenous demand shock in one market can affect the price in another market via hedgers, who trade in multiple markets,

Lemma 3. *The trading equilibrium price vector* $\hat{p}_t$ *is*

$$\hat{p}_t = -\gamma N^{-1}(HN^{-1} + D^{\mathsf{T}}D)^{-1}D^{\mathsf{T}}E_t$$

if all markets have some investor entry. Market shallowness is then

$$\gamma N^{-1}(HN^{-1} + D^{\mathsf{T}}D)^{-1}D^{\mathsf{T}}D.$$

If D^TD is nonsingular (no redundant contracts), the matrix inverse of market shallowness, $\gamma^{-1}[N + H(D^{\mathsf{T}}D)^{-1}]$, measures market depth. The diagonal terms $\partial u_{it}/\partial p_{it}$ are depths for individual markets. In general, individual market depth depends on the number of traders in that market and on the presence of nearby markets. However, for the special case where contract D_i, is orthogonal to all other contracts, individual market depth i reduces to $(H + n_i)/\gamma$. Market i's depth is then proportional to the number of traders in market i; there is no price interaction with other markets.

Consider the entry decision for investors. Define the $r \times r$ symmetric matrix

$$S = \sum_{t=1}^{T} E_t E_t^{\mathsf{T}}$$

as the *net hedging demand.* Assume rank $(S) = r$, so the entire uncertainty space is relevant with respect to aggregate risk. Lemma 4 states

the certainty equivalent value of an investor who enters and trades in one market for T periods.

Lemma 4. *In the trading equilibrium for the market structure* $\{(D_i, n_i) : 1 \le i \le m\}$ *the certainty equivalent value of trading for an investor in market i is*

$$\text{CEV}_i = \frac{\gamma \sum_{t=1}^{T} (D_i^T G_i E_t)^2}{2(H + n_i D_i^T G_i D_i)^2} = \frac{\gamma D_i^T G_i S G_i D_i}{2(H + n_i D_i^T G_i D_i)^2}, \tag{4}$$

where

$$G_i = \left(I + \sum_{j \ne i} \frac{n_j}{H} D_j D_j^T \right)^{-1}, \tag{5}$$

or, equivalently,

$$G_i = I - D_{-i}(HN_{-i}^{-1} + D_{-i}^T D_{-i})^{-1} D_{-i}^T, \tag{6}$$

and D_{-i} is the $r \times (m - 1)$ matrix obtained by deleting column D_i from D.

The matrix G_i, which affects market i investors, captures the impact of the other $(m$ - 1$)$ markets. Multiplying net hedging demand by G_i can be interpreted as factoring out that part of the demand that can be satisfied by using other markets. For instance, if all other markets somehow had unlimited risk-sharing capability $(N_{-i}^{-1} = 0)$, Equation (6) shows that G_i would annihilate the subspace spanned by D_{-i}. Hedging demand within this subspace would be completely satisfied and market i would face zero demand for hedging within this subspace. Generally, since investor entry is limited, this hedging demand is not completely factored out. Since the matrix G_i factors out the risk already hedgeable through other markets, G_i will be called the *factoring matrix*.

Since S represents the net hedging demand across periods, and G_i factors out demand already satisfied, the numerator of (4) measures the amount of hedging satisfied by contract D_i. The more hedging demand satisfied by contract D_i, the higher the certainty value of trading for a market i investor. Therefore, more investors will tend to enter this market: liquidity flows to markets that are good hedges.

3. Exchanges

Lemmas 3 and 4 characterize the trading equilibrium, taking the market structure (contract pairs) as given. Market structure is now endogenized by introducing optimizing exchanges. Assume there are m exchanges, each able to open one market. Each exchange i is

allowed to select a contract D_i and an entering number of investors n_i. An exchange may collect a (membership) fee k_i from each investor entering its market. The maximum fee collectible per investor is given by the investor's certainty value of trading (4) from the trading equilibrium. Exchanges are assumed to maximize the total fees collected in their market, recognizing the market structure of the other markets $\{(D_j, n_j) : j \neq i\}$. ' The optimal innovation (contract pair) problem for a single exchange is first solved; the optimal innovation by multiple exchanges is then considered.

Optimal innovation by a single exchange requires selecting a contract D_i, and a number of entering investors n_i to maximize exchange revenue, recognizing trading equilibrium condition (4) and taking the remaining market structure as fixed. The effect of the remaining market structure is fully captured in the factoring matrix G_i. The innovation problem for exchange i is

$$\max_{D_i, n_i} \pi_i = k_i n_i,$$

$$\text{s.t.} \quad D_i^\mathsf{T} D_i = 1, \qquad k_i = \frac{\gamma}{2} \frac{D_i^\mathsf{T} G_i S G_i D_i}{(H + n_i D_i^\mathsf{T} G_i D_i)^2} \qquad (7)$$

Optimal innovation by a single exchange is characterized in the following proposition.

Proposition 1. *The solution to a single exchange's optimal innovation problem (7) is $(\hat{D}_i, \hat{n}_i)$, where $\hat{D}_i$ is the maximal unit eigenvector of SG_i, and λ_i is its associated eigenvalue. Also,*

$$\hat{n}_i = \frac{H}{\hat{D}_i^\mathsf{T} G_i \hat{D}_i},$$

$$\hat{k}_i = \frac{\gamma}{8H^2} \hat{D}_i^\mathsf{T} G_i S G_i \hat{D}_i = \frac{\gamma \hat{\lambda}_i}{8H^2} \hat{D}_i^\mathsf{T} G_i \hat{D}_i,$$

$$\hat{\pi}_i = \frac{\gamma \hat{\lambda}_i}{8H}. \qquad (8)$$

(The maximal eigenvector is the eigenvector with the greatest eigenvalue.) If SG_i, has a unique maximal eigenvector, then the solution to (7) is unique.

The net hedging-demand matrix S measures desired hedging over both risk space and time. Since $S = \Sigma_t E_t E_t^\mathsf{T}$, net hedging demand is decomposable into a sum (over time) of least-squares projections of single-period demands. Multiplying by G_i factors out hedging demand already provided by other markets. Therefore, SG_i represents unsatisfied, or residual, hedging demand. Selecting the maximal eigenvector

of SG_i amounts to aligning the contract D_i with the greatest residual hedging demand.

The optimal values of n_i and k_i given by Proposition 1 illustrate the nature of interaction between markets. From Equation (5) or (6), note that $D_i^T G_i D_i$ tends to be small when other exchanges have similar contracts ($D_i^T D_j$ near 1) with substantial depth (n_j large). For an exchange whose contract faces such competition ($D_i^T G_i D_i$ is small), the optimal response is to make sure that its contract also offers depth by selecting a large n_i. Of course, this means that the fee k_i must be low. In contrast, the optimal response of an exchange whose contract faces little competition ($D_i^T G_i D_i$ near 1) is to obtain high fees by selecting a small n_i in an exercise of (local) monopoly power.

When selecting a contract, an exchange aligns itself with the greatest unsatisfied hedging demand (maximal eigenvector of SG_i). An exchange thus desires a contract with substantial hedging demand that is not already satisfied by existing contracts. Each exchange tries to locate its contract in its own niche, so revenues do not suffer because of competition from nearby contracts.

Net hedging demand S depends on aggregate endowments. Hedgers who could offset one another's positions (thus providing liquidity and risk sharing for each other) "cancel out" when endowments are aggregated, and do not affect S. Since hedgers only enter exchanges' decision making through S, this demand generates no need for additional investors and goes unrecognized by exchanges. For this reason, an exchange's optimal innovation is not necessarily Pareto-optimal, as the following example illustrates.

Example 1 *(Pareto inefficiency).* There are two hedgers, each of unit mass ($H = 2$), one period ($T = 1$), and one exchange ($m = 1$). A hedger's certainty equivalent value of trading can be calculated (similar to Lemma 4):

$$\text{CEV}_b = \frac{\gamma}{2}\left(D_1^T E_{b1} - \frac{D_1^T E_1}{n_1 + H}\right)^2 = \frac{\gamma}{2}\left(D_1^T E_{b1} - \frac{D_1^T E_1}{4}\right)^2,$$

since $\hat{n}_1 = H = 2$. Exchange revenues are $\hat{\pi}_1 = \gamma \hat{\lambda}_1 / 16$. Let individual endowments be

$$E_{11} = \begin{pmatrix} \alpha \\ \epsilon \end{pmatrix}, \qquad E_{21} = \begin{pmatrix} -\alpha \\ \epsilon \end{pmatrix};$$

hence, aggregate endowment E_1 and net hedging demand S are

$$E_1 = \begin{pmatrix} 0 \\ 2\epsilon \end{pmatrix}, \qquad S = \begin{pmatrix} 0 & 0 \\ 0 & 4\epsilon^2 \end{pmatrix}.$$

[Although S is not of full rank (a technical requirement), adding a

second period could correct this at the cost of obscuring the intuition.] The optimal contract for the exchange is clearly

$$\hat{D}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Also $\overline{\text{CEV}}_1 = \overline{\text{CEV}}_2 = \gamma\epsilon^2/8$ and $\hat{\pi}_1 = \gamma\epsilon^2/4$. If the exchange instead chooses contract

$$D_1' = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

then hedgers receive $\text{CEV}_1' = \text{CEV}_2' = \gamma\alpha^2/2$, although $\pi_1' = 0$. For $\alpha^2 > \frac{3}{4}\epsilon^2$, contract D_1' is Pareto-superior to $\hat{D}_1$, even if the exchange distributes collected revenues to the hedgers.

In this example, for large α, the hedgers would like to trade contract D_1'. The exchanges have no incentive to open such a market, however, since there is no need for investor entry. Thus, the exchange selects a contract that yields less overall risk sharing.

This result contrasts with Duffie and Jackson (1989), who find Pareto optimality with a single exchange and a single period. The difference is due to different sources of exchange revenue: revenue here is collected from investors; revenue in Duffie and Jackson is collected from hedgers. In Duffie and Jackson, inefficiency arises in multiple trading periods because of trading turnover; here, inefficiency arises because the need for risk sharers need not coincide with the need for hedging.

4. Simultaneous innovation

I now consider a game with simultaneous choice of contract pairs by m exchanges. As before, each exchange maximizes total investor fees, taking the rest of the market structure as given. I take Nash equilibrium as the solution concept.

Proposition 2. *An equilibrium exists in the simultaneous innovation game. Optimal contract pairs $(D_i, \hat{n}_i)$ are characterized by Proposition 1.*

The proof of equilibrium is constructive. Define

$$G = \left(I + \sum_{i=1}^{m} \frac{n_i}{H} D_i D_i^{\mathsf{T}} \right)^{-1},$$

which can be interpreted as the factoring matrix an $(m + 1)$st (shadow) exchange would face. The equilibrium constructed has the property

that *SG,* restricted to the span of *D,* is proportional to the identity matrix. Of course, *SG* is identical to *S* off the span. An $(m + 1)$st exchange would therefore be indifferent between all contracts in this span. This particular equilibrium is symmetric, then, in the sense that unfilled hedging demand (over hedges actually served) is equal across risk space.

Equilibrium need not be unique in the simultaneous innovation game. Example 2, for instance, considers a simultaneous innovation game with two exchanges where, depending on the parameters, there may or may not be multiple equilibria. Note that all information about hedgers relevant for equilibrium is captured in the matrix *S.* Without loss of generality, *S* can be taken to be diagonal (this amounts to a change of basis).

Example 2 (two exchanges; simultaneous innovation). There are two exchanges $(m = 2)$. Let

$$S = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix},$$

with $a > b > c > 0$. We consider two cases that differ in the variability of aggregate endowments through time.

Case 1. $a > 2b$. There is a unique equilibrium (up to relabeling exchanges and multiplying contracts by - 1). The contracts chosen are

$$D_1 = \frac{1}{\sqrt{a^2 + b^2}} \begin{pmatrix} a \\ b \\ 0 \end{pmatrix}, \qquad D_2 = \frac{1}{\sqrt{a^2 + b^2}} \begin{pmatrix} a \\ -b \\ 0 \end{pmatrix},$$

with eigenvalues $\lambda_1 = \lambda_2 = 2ab/(a + b)$. Investor entry is

$$n_1 = n_2 = [(a^2 + b^2)/2ab]H.$$

The matrices SG_i, are

$$SG_1 = \begin{pmatrix} \zeta(2a + b) & \zeta a & 0 \\ \zeta b & \zeta(a + 2b) & 0 \\ 0 & 0 & c \end{pmatrix},$$

$$SG_2 = \begin{pmatrix} \zeta(2a + b) & -\zeta a & 0 \\ -\zeta b & \zeta(a + 2b) & 0 \\ 0 & 0 & c \end{pmatrix},$$

where $\zeta = ab/(a + b)^2$. It is easy to check that D_1 and D_2 are maximal eigenvectors with respect to SG_1 and SG_2.

Case 2. $a \le 2b$. There are two equilibria: the first is described in case 1; the second takes the form

$$D_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \qquad D_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},$$

with eigenvalues $\lambda_1 = a$ and $\lambda_2 = b$. Investor entry is $n_1 = n_2 = H$. The matrices SG_i are

$$SG_1 = \begin{pmatrix} a & 0 & 0 \\ 0 & b/2 & 0 \\ 0 & 0 & c \end{pmatrix}, \qquad SG_2 = \begin{pmatrix} a/2 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix},$$

and D_1 and D_2 are clearly maximal eigenvectors.

Since there are only two exchanges, the third dimension of risk space, with the least net hedging demand, is ignored. The equilibrium of case 1 is symmetric: the two contracts are comparably aligned to the net hedging demand, and exchange profits (proportional to λ_i) are equal. This is the previously mentioned constructible equilibrium. The equilibrium of case 2 is asymmetric: each exchange serves a different dimension of risk, and exchange profits are unequal.

I next examine the nature of competition between exchanges, focusing first on any two exchanges (in a simultaneous innovation game with $m \geq 2$). The similarity (high correlation) of two contracts implies competition through liquidity, in the sense that the two exchanges increase investor entry in order to increase their market share. The following proposition gives a lower bound on investor entry.

Proposition 3. *For any two markets i and j in a simultaneous innovation equilibrium,*

$$\hat{n}_i \geq \frac{H}{\sqrt{1 - \text{Corr}^2(i, j)}},$$

where the correlation of the payoffs offutures contracts i and j is

$$\text{Corr}(i, j) = D_i^{\mathsf{T}} D_j.$$

If there are only two exchanges, the inequality holds with equality.

By calculating an upper bound on investor entry, one gets an upper bound on similarity of contracts.

Corollary. *For any two markets i and j in a simultaneous innovation equilibrium,*

$$\text{Corr}^2(i, j) \leq 1 - \left(\frac{\lambda_s^-}{\lambda_s^+}\right)^2,$$

where λ_S^+ and λ_S^- are the largest and smallest eigenvalues of S, respectively.

Corollary. *In a simultaneous innovation equilibrium, no two exchanges choose the same contract.*

This illustrates the nature of the competition for the hedger's business. If contracts are highly correlated, exchanges compete for the same hedging demand. Exchanges compete by offering more liquidity in the form of more investors at their market. To accomplish this necessitates settling for lower per-investor fees; thus competition drives down overall revenues for an exchange. In the extreme, if two exchanges selected identical contracts, the resulting Bertrand-like competition in the size of the fee the exchange is willing to accept drives profits to zero. Beyond a certain point, each exchange could do better by selecting a less-contested contract with less hedging demand to begin with. Thus, in equilibrium, each exchange finds its own niche (a relatively uncontested contract) to be able to exercise some monopoly power (in selecting investor entry and, implicitly, fees).

Proposition 3 and its corollaries show that no two contracts are too highly correlated. I consider the entire market structure as a whole in Proposition 4 and achieve a similar result.

Proposition 4. *In a simultaneous innovation equilibrium,* rank $(\hat{D})$ $> m/2$.

The intuition of this result is similar to that of the corollaries to Proposition 3. If m exchanges try to locate their contracts in a subspace of dimension no more than $m/2$, then, loosely speaking, the "average" dimension of risk space has at least two contracts serving it. With two exchanges per dimension, exchanges are in Bertrand-like competition over setting (implicit) fees; fees and revenues are then driven to zero. Exchanges have an incentive to switch to a contract in an uncontested dimension of risk space. This incentive is present as long as rank$(D) \leq m/2$. Again, in equilibrium, each exchange selects a contract to allow itself some monopoly power in selecting investor entry.

5. Sequential Innovation

I next consider a game with sequential choice of contract pairs by exchanges. Additionally, a fixed cost of design and innovation is imposed on each exchange that chooses to open a market. Therefore, some exchanges may decide not to innovate. In equilibrium, contracts

are orthogonal, and exchanges provide sufficient liquidity in their market to preempt potential competitors.

The number of exchanges is potentially unlimited (although no more than renter in equilibrium). Each exchange, in order, decides whether to innovate, thus incurring a fixed (setup or design) cost C, before any trade takes place. An exchange that decides to innovate selects a contract D_i, and a number of investors n_i entering its market (thus implicitly setting a fee k_i). An exchange maximizes its net revenues $\pi_i = k_i n_i - C$ (total investor fees minus innovation cost), recognizing both previous innovations and the possibility of subsequent innovations. (I adopt the convention that exchanges only innovate if they receive strictly positive net revenues.) Thus, subgame perfection is the solution concept. Proposition 1 does not characterize an optimal innovation in this context, since the behavior of exchanges that have yet to move (contained in G_i) cannot be taken as given. Optimal innovation in this setting must recognize the sequential nature of the game and the ability of early innovators to affect later innovators. Proposition **5** characterizes the sequential innovation equilibrium.

Proposition 5. *An equilibrium exists in the sequential innovation game. Contracts chosen (D_i) are the eigenvectors of S with eigenvalues $\lambda_i > 8HC/\gamma$.*

If $\lambda_i \geq 16HC/\gamma$, then

$$\hat{n}_i = \gamma\lambda_i/8C - H, \qquad \hat{\pi}_i = 3C - 32HC^2/\gamma\lambda_i.$$

If $8HC/\gamma < \lambda_i \leq 16HC/\gamma$, then

$$\hat{n}_i = H, \qquad \hat{\pi}_i = \gamma\lambda_i/8H - C.$$

Futures contracts chosen are orthogonal. If the eigenvectors of S are distinct, equilibrium is unique.

In sequential innovation equilibrium, the first exchange to move selects the contract that aligns with the highest net hedging demand (D_1 the maximal eigenvector of S). Investor entry is chosen to be large enough to satisfy most of the hedging demand in the direction D_1. More precisely, residual hedging demand in direction D_i is made so small that an exchange opening an identical contract cannot collect enough revenues to cover its fixed cost. If the first exchange faces low hedging demand $(\lambda_1 \leq 16HC/\gamma)$, the monopolistic level of investor entry $(n_1 = H)$ is enough to deter competitors. If it faces high hedging demand $(\lambda_1 > 16HC/\gamma)$, it would still like to exercise monopoly power to keep fees high. The threat of potential competition prevents this exchange from doing so. To deter competitors,

the exchange increases investor entry to $n_1 = \gamma\lambda_1/8C - H$. This guarantees that no other exchange will find it profitable to design a contract similar to D_1; subsequent contracts will be orthogonal to D_1.

The second exchange to move selects the contract that aligns with the highest unfilled net hedging demand $(D_2$, the second largest eigenvector of $S)$. Again, a sufficiently high level of investor entry n_2, is chosen to preempt other exchanges from selecting the same contract. The process continues with other exchanges until no profitable potential contracts remain.

Since contracts are orthogonal, trade in one market does not affect trade in another. Therefore, the depth of market i is measured simply by $(H + n_i)/\gamma$, as discussed in Section 2. In selecting n_i, exchange i in fact chooses the liquidity (market depth) of its futures contract.

In a sequential innovation equilibrium, the number of markets innovated equals the number of eigenvalues of S greater than $8HC/\gamma$ (which is bounded above by r). Both liquidity and exchange revenues are increasing in net hedging demand served (measured by λ_i). Contracts serving high hedging demand $(\lambda_i \geq 16HC/\gamma)$ have high liquidity (market depth of $\lambda_i/8C$); contracts serving moderate hedging demand $(8HC/\gamma < \lambda_i \leq 16HC/\gamma)$ have minimum liquidity (market depth of $2H/\gamma$) contracts that would serve low hedging demand (X, $\leq 8HC/\gamma)$ are never innovated. Since exchange revenues are increasing in λ_i, exchanges that innovate first have an advantage. I close with an example of a sequential innovation equilibrium.

Example 3 *(sequential innovation).* There is a unit mass of hedgers $(H = 1)$ with risk aversion $\gamma = 1$. Suppose the innovation cost $C = 1$ and

$$S = \begin{pmatrix} 25 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 5 \end{pmatrix}.$$

Two contracts are innovated: D_1 and D_2, where

$$D_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \qquad D_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

Exchange 1 chooses a market depth of $\frac{25}{8}\left(n_1 = \frac{17}{8}\right)$ to deter entry along D_1. Exchange 2 chooses the minimal market depth of 2 $(n_2 = 1)$, more than enough to deter entry along D_2. Exchange 1 makes $\pi_1 = 3 - \frac{32}{25} = 1.72$; exchange 2 makes $\pi_2 = \frac{15}{8} - 1 = 0.875$. Exchange 3 faces a residual hedging demand matrix with eigenvalues of 8, 7.5,

and **5;** it needs an eigenvalue above 8 to obtain a positive profit, so it does not innovate.

6. conclusion

I have presented a model of market innovation in which exchanges compete to be able to share the risk of the hedgers. Market frictions take two forms: (1) exchanges charge a membership fee to investors entering their market, limiting the entry and risk sharing that can take place; and (2) investors are allowed to enter only one market, and their risk-sharing ability is therefore limited.

Exchanges choose both a contract and the number of investors who enter their market (and, implicitly, the entry fee), taking the market structure (contracts and investors in other markets) into account. Exchanges have the objective of maximizing their total revenue collected through fees. Hedgers trade in all markets; investors trade only in their chosen market. Liquidity, the ability of a market to absorb risk, depends on the total number of traders in that market and on the structure of similar markets.

The optimal innovation for a single exchange (the revenue-maximizing contract and investor entry level taking other markets as fixed) is derived. An exchange optimizes in two ways: (1) choosing a contract to fill hedging demand not met by other exchanges, and (2) using the resulting monopoly power to limit investor entry, keeping fees high. Innovations need not be Pareto-optimal, since an exchange considers net hedging demand, while substantial offsetting hedging demand may be present.

A simultaneous innovation game is considered. Equilibrium need not be unique, but a particular equilibrium with symmetric properties is described. Exchanges that select highly correlated contracts would implicitly compete in a Bertrand-like manner over fees, decreasing overall revenues. To avoid this, exchanges try to find a niche by offering dissimilar contracts. In equilibrium, the correlation of the contract payoffs for any two markets is bounded away from unity. Similar contracts exhibit high liquidity in the form of high investor entry.

A sequential innovation game is also considered, where exchanges must incur fixed costs in order to innovate and recognize their effect on later potential innovations. Equilibrium is generically unique. Exchanges that innovate first are at an advantage; they can choose contracts to serve the highest hedging demand. The threat of later innovations with a similar contract forces the exchange to choose a high level of investor entry, preempting potential competitors completely. In equilibrium, contracts offered are orthogonal. Contracts

serving the highest hedging demand offer the highest liquidity, con-
tracts serving moderate hedging demand offer a minimal liquidity,
and contracts that would serve low hedging demand are not profitable
enough to be offered.

Appendix: Proofs

Proof (Lemmas 1, 2). Take first-order conditions and aggregate over
traders. ∎

Proof (Lemma 3). Using Lemmas 1 and 2 and market clearing, one
can write

$$p_t = \gamma N^{-1}(HN^{-1} + D^{\mathsf{T}}D)^{-1}(D^{\mathsf{T}}Du_t - D^{\mathsf{T}}E_t).$$

This immediately yields the market shallowness matrix; setting $u_i = 0$ yields $\hat{p}_t$. ∎

Proof (Lemma 4). Let $B = (HN^{-1} + D^{\mathsf{T}}D)$, and let $(B^{-1})_i$ be the ith
row of B^{-1}. Substitute from Lemmas 2 and 3 into problem (2) to get

$$\mathrm{CEV}_i = \frac{1}{2\gamma} \sum_{t=1}^{T} p_{it}^2 = \frac{\gamma}{2n_i^2} \sum_{t=1}^{T} [(B^{-1})_i^{\mathsf{T}} D^{\mathsf{T}} E_t]^2.$$

Write B as

$$B = \begin{pmatrix} (n_i + H)/n_i & D_i^{\mathsf{T}} D_{-i} \\ D_{-i}^{\mathsf{T}} D_i & B_{-i} \end{pmatrix},$$

where B_{-i} is the $(m - 1) \times (m - 1)$ matrix remaining after deleting
the ith row and ith column of B, and D_{-i} is the $r \times (m - 1)$ matrix
remaining after deleting the ith column of D. Define the matrix $G_i = I - D_{-i}(B_{-i})^{-1}D_{-i}^{\mathsf{T}}$. It can be shown by direct calculation that

$$B^{-1} = \begin{pmatrix} \beta & -\beta D_i^{\mathsf{T}} D_{-i}(B_{-i})^{-1} \\ -\beta(B_{-i})^{-1}D_{-i}^{\mathsf{T}}D_i & (B_{-i} - [n_i/(n_i + H)] D_{-i}^{\mathsf{T}}D_iD_i^{\mathsf{T}}D_{-i})^{-1} \end{pmatrix},$$

where $\beta = (H/n_i + D_i^{\mathsf{T}}G_iD_i)^-$ and the ith row and ith column have
been put in the first place. Therefore,

$$(B^{-1})_iD^{\mathsf{T}}E_t = \beta D_i^{\mathsf{T}}E_t - \beta D_i^{\mathsf{T}}D_{-i}(B_{-i})^{-1}D_{-i}^{\mathsf{T}}E_t$$

$$= \frac{D_i^{\mathsf{T}}G_iE_t}{H/n_i + D_i^{\mathsf{T}}G_iD_i}.$$

Substitute into the CEV, expression above to get (4). Now

$$G_i = I - D_{-i}(B_{-i})^{-1}D_{-i}^{\mathsf{T}} = I - D_{-i}(HN_{-i}^{-1} + D_{-i}^{\mathsf{T}}D_{-i})_{-i}^{-1}D_{-i}^{\mathsf{T}},$$

which is Equation (6). Since

$$G_i(I + H^{-1}D_{-i}N_{-i}D_{-i}^{\mathrm{T}})$$

$$= I + H^{-1}D_{-i}N_{-i}D_{-i}^{\mathrm{T}} - D_{-i}(HN_{-i}^{-1} + D_{-i}^{\mathrm{T}}D_{-i})^{-1}D_{-i}^{\mathrm{T}}$$

$$\quad - H^{-1}D_{-i}(HN_{-i}^{-1} + D_{-i}^{\mathrm{T}}D_{-i})^{-1}D_{-i}^{\mathrm{T}}D_{-i}N_{-i}D_{-i}^{\mathrm{T}}$$

$$= I + H^{-1}D_{-i}[I - (HN_{-i}^{-1} + D_{-i}^{\mathrm{T}}D_{-i})^{-1}(HN_{-i}^{-1} + D_{-i}^{\mathrm{T}}D_{-i})]N_{-i}D_{-i}^{\mathrm{T}}$$

$$= I,$$

it follows that

$$G_i = (I + H^{-1}D_{-i}N_{-i}D_{-i}^{\mathrm{T}})^{-1} = \left(I + \frac{1}{H}\sum_{j \neq i} n_j D_j D_j^{\mathrm{T}}\right)^{-1},$$

which is Equation (5). ∎

Proof (Proposition 1). First maximize

$$\pi_i = \frac{\gamma n_i(D_i^{\mathrm{T}} G_i SG_i D_i)}{2(H + n_i D_i^{\mathrm{T}} G_i D_i)^2}$$

over n_i. The first-order condition yields $\hat{n}_i = H/D_i^{\mathrm{T}} G_i D_i$; substituting back reduces optimization (7) to

$$\max_{D_i} \frac{\gamma}{8H} \frac{D_i^{\mathrm{T}} G_i SG_i D_i}{D_i^{\mathrm{T}} G_i D_i},$$

$$\text{s.t.} \quad D_i^{\mathrm{T}} D_i = 1.$$

If we temporarily disregard the (unit-normalizing) constraint, the first-order condition on D_i implies

$$(D_i^{\mathrm{T}} G_i D_i) G_i SG_i D_i = (D_i^{\mathrm{T}} G_i SG_i D_i) G_i D_i.$$

Premultiply by G_i^{-1} to get

$$SG_i D_i = \frac{D_i^{\mathrm{T}} G_i SG_i D_i}{D_i^{\mathrm{T}} G_i D_i} D_i.$$

Therefore, D_i is an eigenvector of SG_i with eigenvalue $\lambda = D_i^{\mathrm{T}} G_i SG_i D_i / D_i^{\mathrm{T}} G_i D_i$. Since this is the quantity being maximized, choose the largest eigenvalue. Finally, normalize D_i to be a unit vector. ∎

Proof (Proposition 2). Available from the author on request. ∎

Proof (Proposition 3). Let $\rho = \mathrm{Corr}(i, j)$. In equilibrium,

$$\frac{H}{n_i} = D_i^{\mathrm{T}} G_i D_i \leq D_i^{\mathrm{T}}\left(I + \frac{n_j}{H} D_j D_j^{\mathrm{T}}\right)^{-1} D_i$$

$$= D_i^{\mathrm{T}}\left(I - \frac{n_j}{H + n_j} D_j D_j^{\mathrm{T}}\right) D_i = \frac{H + (1 - \rho)^2 n_j}{H + n_j}.$$

Cross-multiplying and using symmetry give

$$H^2 + Hn_j \le Hn_i + (1 - \rho^2) n_i n_j,$$

$$H^2 + Hn_i \le Hn_j + (1 - \rho^2) n_i n_j.$$

Therefore, $H^2 \le (1 - \rho^2) n_i n_j$. (For only two exchanges, the inequalities above are equalities.) Since $n_j - (1 - \rho^2) n_i n_j \le Hn_i - H^2$, if $n_i < H/\sqrt{1 - \rho^2}$, then

$$n_j \le \frac{Hn_i - H^2}{H - (1 - \rho^2) n_i} < \frac{H^2/\sqrt{(1 - \rho^2)} - H^2}{H - H\sqrt{1 - \rho^2}} = \frac{H}{\sqrt{1 - \rho^2}}.$$

Therefore, $n_i n_j < H^2/(1 - \rho^2)$, a contradiction. ∎

Proof (Corollaries). Note $n_i = H/D_i^{\mathsf T} G_i D_i$. In equilibrium,

$$D_i^{\mathsf T} G_i D_i = D_i^{\mathsf T} S^{-1} SG_i D_i = \hat\lambda_i D_i^{\mathsf T} S^{-1} D_i \ge (\lambda_{\bar s})(\lambda_{\bar s}^+)^{-1},$$

since $\lambda_{\bar s}$ is a lower bound on $\hat\lambda_i$ and $(\lambda_{\bar s}^+)^{-1}$ is the smallest eigenvalue of S^{-1}. Noting that

$$\mathrm{Corr}^2(i, j) \le 1 - H^2/n_i^2 = 1 - (D_i^{\mathsf T} G_i D_i)^2$$

proves the first corollary; noting that $\lambda_{\bar s} > 0$ proves the second. ∎

Proof (Proposition 4). Available from the author on request. ∎

Proof (Proposition 5). For some sequential innovation equilibrium, write

$$G = \left(I + \sum_j \frac{n_j}{H} D_j D_j^{\mathsf T} \right)^{-1}, \qquad \Gamma_i = \left(I + \sum_{j<i} \frac{n_j}{H} D_j D_j^{\mathsf T} \right)^{-1}.$$

Let l be the largest eigenvalue of SG. For a potential additional exchange x,

$$\pi_x \le \frac{\gamma}{2} \frac{n_x D_x^{\mathsf T} GSGD_x}{(H + n_x D_x^{\mathsf T} GD_x)^2} - C$$

$$= \frac{\gamma}{2} \frac{n_x D_x^{\mathsf T} GD_x}{(H + n_x D_x^{\mathsf T} GD_x)^2} \frac{D_x^{\mathsf T} GSGD_x}{D_x^{\mathsf T} GD_x} - C \le \frac{\gamma\lambda}{8H} - C.$$

If $\lambda \le 8HC/\gamma$, then $\pi_x \le 0$ and x will not enter. Conversely, if $\lambda > 8HC/\gamma$, then x will enter.

A potential exchange entering after x faces

$$G_x = \left(G^{-1} + \frac{n_x}{H} D_x D_x^{\mathsf T} \right)^{-1} = G - \frac{n_x}{H + n_x D_x^{\mathsf T} GD_x} GD_x D_x^{\mathsf T} G.$$

Choosing D_x as the maximal eigenvector of SG can deter further entry and be profitable for x. If $\lambda \le 16HC/\gamma$, take $n_x = H/D_x^{\mathsf T} GD_x$; then

$D_x^T SG_x D_x = \lambda/2 \leq 8HC/\gamma$, and $\pi_x = \gamma\lambda/8H - C > 0$. If $\lambda \geq 16HC/\gamma$, take $n_x = (\gamma\lambda/8C - H)/D_x^T GD_x$; then $D_x^T SG_x D_x = 8HC/\gamma$, and $\pi_x = 3C - 32HC^2/\gamma\lambda \geq C >$! Thus, a contract D_o can be profitable for the exchange moving after i iff $D_0^T S\Gamma_{i+1} D_0 > 8HC/\gamma$.

I now show, via induction on i, that exchanges open markets, in order, along the largest eigenvectors of S (for eigenvalues $> 8HC/\gamma$) and select n's to deter future competitors. Exchange i faces

$$\max \pi_i = \frac{\gamma}{2} \frac{n_i D_i^T G_i SG_i D_i}{(H + n_i D_i^T G_i D_i)^2} - C$$

subject to maximum eigenvalue $(SG) \leq 8HC/\gamma$. There are two cases.

Case 1. If the threat of competition is nonbinding, the problem reduces to

$$\max \pi_i = \frac{\gamma}{2} \frac{n_i D_i^T \Gamma_i D_i}{(H + n_i D_i^T \Gamma_i D_i)^2} \frac{D_i^T \Gamma_i S\Gamma_i D_i}{D_i^T \Gamma_i D_i} - C.$$

From the proof of Proposition 1, take $\hat{D}_i$ to be the maximal eigenvector of $S\Gamma_i$ (with eigenvalue λ_i) and $\hat{n}_i = H/\hat{D}_i^T \Gamma_i \hat{D}_i$. The threat is nonbinding iff $D_i^T S\Gamma_{i+1} D_i = \lambda_i/2 \leq 8HC/\gamma$, or $\lambda_i \leq 16HC/\gamma$.

Case 2. If the threat of competition is binding,

$$\frac{8HC}{\gamma} = D_i^T SGD_i \leq \lambda_i D_i^T GD_i \leq \lambda_i D_i^T \Gamma_{i+1} D_i = \frac{H\lambda_i}{H + n_i},$$

so $n_i \leq \gamma\lambda_i/8C - H$. By substituting

$$G_i = G + \frac{n_i}{H - n_i D_i^T GD_i} GD_i D_i^T G$$

into π_i,

$$\pi_i = \frac{\gamma}{2H^2} (n_i D_i^T GD_i) \frac{D_i^T GSGD_i}{D_i^T GD_i} - C.$$

Since

$$\frac{D_i^T GSGD_i}{D_i^T GD_i} \leq \frac{8HC}{\gamma} \quad \text{and} \quad n_i D_i^T GD_i \leq n_i D_i^T \Gamma_{i+1} D_i = \frac{Hn_i}{H + n_i}$$

(increasing in n_i), if $n_i = \gamma\lambda_i/8C - H$, $D_i^T GD_i = D_i^T \Gamma_{i+1} D_i$, and $D_i^T GSGD_i/D_i^T GD_i = 8HC/\gamma$, the optimum is found. Choose $\hat{D}_i$ to be the maximum eigenvector of $S\Gamma_i$, and $\hat{n}_i = \gamma\lambda_i/8C - H$. Then $D_i^T S\Gamma_{i+1} D_i = 8HC/\gamma$, so entry along D_i is deterred. Therefore, $D_i^T GD_i = D_i^T \Gamma_{i+1} D_i$, and

$$\frac{D_i^T GSGD_i}{D_i^T GD_i} = \frac{D_i^T \Gamma_{i+1} S\Gamma_{i+1} D_i}{D_i^T \Gamma_{i+1} D_i} = \frac{H\lambda_i}{H + n_i} = \frac{8HC}{\gamma}.$$

Exchange 1 sets D_1 to the largest eigenvector of S and chooses $n_1 \leq 8HC/\gamma$;exchange 2 then opens D_2 along the largest eigenvector of $S\Gamma_1$ (the second largest eigenvector of S), continuing until all eigenvalues(S) $> 8HC/\gamma$ are used. ∎

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